ICMAT
INSTITUTO DE CIENCIAS MATEMÁTICAS

EXCELENCIA SEVERO OCHOA

Conference in honor of Javier Cilleruelo

THE MUSIC OF NUMBERS

Instituto de Ciencias Matemáticas
20 - 22 September, 2017
Madrid

The conference will be dedicated to the memory of Javier Cilleruelo, who passed away last year. Our aim is to celebrate his mathematical and personal legacy, by bringing together leading researchers working on the topics that were close to Prof. Cilleruelo's mathematical interests.

As such, the scope of the conference encompasses topics in analytic, additive and combinatorial number theory, as well as related fields.

Organizers:

Pablo Candela
(ICMAT-UAM)
Antonio Córdoba
(ICMAT-UAM)
Gonzalo Fiz
(UPC-BGSMath)
Juanjo Rué
(UPC-BGSMath)
Oriol Serra
(UPC-BGSMath)
Ana Zumalacárregui
(UNSW)

More information: http://musicofnumbers.kissr.com/

UAM UNIVERSIDAD AUTONOMA DE MADRID

DEPARTAMENTO DE MATEMÁTICAS
UNIVERSIDAD AUTÓNOMA DE MADRID

List of Speakers

- Antonio Córdoba (U. Autónoma de Madrid - I. Ciencias Matemáticas).
- Jean-Marc Deshouillers (U. Bordeaux).
- Pablo Fernández (U. Autónoma de Madrid).
- Gonzalo Fiz-Pontiveros (U. Politècnica de Catalunya).
- Harald Helfgott (U. Göttingen and I. Mathematics Jussieu).
- Jorge Jiménez (U. Politècnica de Catalunya).
- Sandor Kiss (Budapest U. of Technology and Economics).
- Florian Luca (U. Witwatersrand).
- Máté Matolsci (Rényi Institute).
- Alain Plagne (É. Polytechnique).
- Sean Prendiville (U. of Manchester).
- Surya Ramana (Harish-Chandra Research I.).
- Olivier Ramaré (Institut de Mathématiques de Marseille).
- Oliver Roche-Newton (Johannes Kepler Universität).
- Misha Rudnev (University of Bristol).
- Manuel Silva (U. Nova de Lisboa).
- Christoph Spiegel (U. Politècnica de Catalunya).
- Lluis Vena (U. of Amsterdam).
- Julia Wolf (University of Bristol).

	Wednesday 20th	Thursday 21st	Friday 22nd
10:00-11:00	**Córdoba**	**Helfgott**	**Deshouillers**
11:00-11:30	**Coffee Break**	**Coffee Break**	**Coffee Break**
11:30-12:00	**Rudnev**	**Roche-Newton**	**Wolf**
12:00-12:30	**Prendiville**	**Plagne**	**Luca**
12:30-14:30	**LUNCH**	**LUNCH**	**LUNCH**
14:30-15:00	**Spiegel**	**Fiz-Pontiveros**	**Kiss**
15:00-15:30	**Coffee Break**	**Coffee Break**	**Coffee Break**
15:30-16:00	**Jiménez**	**Silva**	**Vena**
16:00-16:30	**Matolcsi**	**Ramaré**	**Fernández**
16:30-16:45	**Break**	**Break**	
16:45-17:15	**Ramana**	**Vinuesa**	
17:15-17:30		**Break**	
17:30-18:30		**Problem Session**	
19:30-22:00		**Dinner at El Goloso**	

Useful directions

The conference will take place in the ICMAT (Instituto de Ciencias Matemáticas) building, located in the campus of Universidad Autónoma de Madrid. All talks will be held in Aula Naranja. Registration will be open from 9 am on Wednesday 20th.

WIFI DETAILS:

network: ICMAT   password: mgvem3488

The simplest way to get to the institute is by RENFE Cercanias train (line C4, direction Alcobendas or Colmenar Viejo). Get off the train at Cantoblanco Universidad station and head straight down Francisco Tomás y Valiente street. The ICMAT is at the end of this street, just on the intersection with Nicolás Cabrera street (see directions on the map below).

There will be coffee breaks at the institute mornings and afternoons. Lunch is also included: a two course meal and dessert served in Plaza Mayor, 10 min away from the institute (number 26 on the map).

The conference dinner will take place on Thursday at 19:30 in the restaurant El Goloso, also located inside the campus (number 26 on the map).

WEDNESDAY 20th September

The music of numbers

Lattice points on small arcs: remarks and open problems in the work of Javier Cilleruelo

Antonio Córdoba (U. Autónoma de Madrid - ICMAT)

Abstract: How many lattice points can be placed on an arc of length R^α on the circle $|x| = R$? This is an interesting question about which Javier Cilleruelo obtained several remarkable results. In this talk we shall consider its connections with some important developments in modern Fourier analysis, pointing out several open problems.

Sum-products, bilinear forms, cross-ratios...

Misha Rudnev (University of Bristol)

Abstract: It appears that in the sum-product theory two open questions stand out as somewhat pivotal. One is: given a finite plane point set P and a nondegenerate bilinear form B, what is the minimum size of the set of values of B on pairs of points of P? The other – what is the minimum number of distinct cross-ratios, determined by a finite set A in a field K, sufficiently small if K has positive characteristic p; the same concerns the above set P if the plane is over K.

The connection between the two questions arises when B is skew-symmetric. The talk skims over some state of the art bounds and open questions.

Partition regularity of certain non-linear Diophantine equations

Sean Prendiville (U. of Manchester)

Abstract: We survey some results in additive Ramsey theory which remain valid when variables are restricted to sparse sets of arithmetic interest, in particular the partition regularity of a class of non-linear Diophantine equations in many variables.

Random Strategies are Nearly Optimal for Rado Games

Christoph Spiegel (U. Politècnica de Catalunya)

Abstract: Beck introduced the *van der Waerden games* as the Maker-Breaker positional games in which Maker attempts to occupy an arithmetic progression of given length in $[n] = \{1, \dots, n\}$. We study the biased version of a strong generalization of this class of games, considering solutions to any abundant, homogeneous system. We determine the threshold biases of them up to constant factors by proving general winning criteria for Maker and Breaker based on the ideas developed by Bednarska and Łuczak. These general criteria also allow us to easily study the hypergraph generalization of the biased H-games. They also show that a random strategy for Maker is the best known strategy, giving further insight into the so-called *probabilistic intuition* in positional games by indicating a strong connection to sparse Turán- and Szemerédi-type statements.

Irreducibility of polynomials in two variables

Jorge Jiménez Urroz (Universidad Politècnica de Catalunya)

Abstract: In order to control the spectrum of singularities of the generalized Riemann function, we need to find sharp bounds for certain kind of Gauss sums with frequencies on polynomials. This can be achieved by Weil bounds on completely irreducible algebraic curves, which lead us to prove some theorems on irreducibility of polynomials in two variables. We will prove a general theorem in this field. An example of the theorem is the absolute irreducibility of $p(x) - p(y) + 1$, for any $p(x)$ with integer coefficients.

Bounds on the supremum of autoconvolutions

Máté Matolsci (Rényi Institute)

Abstract: Javier Cilleruelo in a joint work with Imre Ruzsa and Carlos Vinuesa established a connection between the (asymptotic) maximal size of g-Sidon sets and the following problem on autoconvolutions of functions: given a nonnegative function f on the interval [-1/4, 1/4], with integral 1, what is the minimal possible value of the supremum of the autoconvolution $f * f$? I will discuss the best known bounds for the latter problem (joint work with Carlos Vinuesa).

Quotients and Products of Thin Subsets of the Positive Integers

Surya Ramana (Harish-Chandra Research I.)

Abstract : This talk is about two papers with Javier Cilleruelo and Olivier Ramaré. The first is a paper titled "Quotients and products of thin subsets of the positive integers" in Proc. Steklov Inst. of Math., 2017 and the second is the paper "The number of rational numbers determined by large sets of integers." in Bull. Lond. Math. Soc., 2010. The first paper is a wider investigation on the theme of the second. The result at the base of these papers is an essentially optimal upper bound for the number of rational numbers that can be written as $\frac{a}{b}$, with $(a, b) \in A \times B$, where $A \subset \{1, \ldots, N\}$ has density at least α and $B \subset \{1, \ldots, M\}$ has density at least β.

THURSDAY 21st of September

The music of numbers[1]

The natural quadratic sieve

Harald Helfgott (U. Göttingen and I. Mathematics Jussieu).

Abstract: Sums involving the Moebius function arise naturally all over analytic number theory. A sieve is, almost by definition, a weighted sum of this kind, but weighted sums can appear in other contexts, as if of their own will, whether one is using combinatorial identities - such as Vaughan's, or studying the zeta function, say.
Sums of the form

$$\sum_m \left(\sum \mu(d)\rho(d)\right)^2,$$

where rho is a function of restricted support, are familiar thanks in part to Selberg's sieve, which is precisely such a sum in which rho has been optimized as a function on the intergers. However, in many contexts, a smooth rho is preferrable - or forced on us by the context in which the sum arises.
A particular case - that of a smooth rho proportional to log as it transitions between two constant functions - was studied by Barban, Vehov and Graham, among others. In particular, it was known that its leading term was as small, asymptotically, as that of the Selberg sieve.
We shall see how a completely explicit treatment shows the true quality of this smooth rho. The sieve thus obtained is practical even for relatively small integers.

Iterated sum-product results

Oliver Roche-Newton (Johannes Kepler Universität)

Abstract: This talk discusses ongoing work in which we attempt to use ideas of Bourgain and Chang on iterated sum-product estimates over the integers to obtain new results. The primary aim is to prove an analogous result concerning iterated product sets and products of shifts.

[1] We will have the social dinner in the evening.

News from Sidon sets

Alain Plagne (É. Polytechnique)

Abstract: We will speak about Sidon sets and their generalizations called $B_h[g]$ sets. Starting with early works, recalling some of Javier's works and concluding with some recent results.

Virtually Fibering Random Right-Angled Coxeter Groups

Gonzalo Fiz Pontiveros (U. Politècnica de Catalunya)

Abstract: A group K *virtually algebraically fibers* if there is a finite index subgroup K' admitting a surjective homomorphism $K' \to \mathbb{Z}$ with finitely generated kernel. The notion arises from topology: a 3-manifold M is virtually a surface bundle over a circle precisely when the fundamental group of M virtually algebraically fibers Recently, Jankiewicz, Norin, and Wise translated the problem virtually fibering a right-angled Coxeter group K into a combinatorial game on an associated graph Γ_K. The talk will discuss sharp thresholds obtained for the virtual fibering of a random right-angled Coxeter group.

Joint work with R. Glebov and I. Karpas.

Ramsey Theory for infinite words

Manuel Silva (U. Nova de Lisboa)

Abstract: In combinatorics of words, a concatenation of k consecutive equal blocks is called a power of order k. We define an anti-power of order k as a concatenation of k consecutive pairwise distinct blocks of the same length. We show that every infinite word contains powers of any order or anti-powers of any order. That is, the existence of powers or anti-powers is an unavoidable regularity.

Products of primes in arithmetic progressions

Olivier Ramaré (Institut de Mathématiques de Marseille)

Abstract: One of Linnik's Theorem asserts that there exist constants c_1 and c_2 such that every arithmetic progressions $\{a + mq, m \in \mathbb{N}\}$ contains a prime $\leq c_1 q^{c_2}$, provided that a be prime to q. After a survey of explicit results related to this theorem, we will explain a recent proof of A. Walker and the author that establishes in an extremely simple way that the above arithmetic progression contains a product of three primes, each being not more than $q^{16/3}$. The talk will end with the presentation of some results and problems concerning products of primes in a given arithmetic progression.

FRIDAY 22nd of September

The music of numbers

Sums of powers, a probabilistic approach

Jean-Marc Deshouillers (U. Bordeaux)

Abstract: Erdős and Rényi introduced in 1960 the study of sums of s-th powers by probability methods. Their model has been largely developed since then. The talk will give a survey of this probabilistic approach, up to the most recent contributions obtained jointly with Javier Cilleruelo.

Szemerédi's Theorem in the primes

Julia Wolf (University of Bristol)

Abstract: Green and Tao famously proved in 2005 that any subset of the primes of fixed positive density contains arbitrarily long arithmetic progressions. Green had previously shown that in fact any subset of the primes of relative density tending to zero sufficiently slowly contains a 3-term term progression. This was followed by work of Helfgott and de Roton, and Naslund, who improved the bounds on the relative density in the case of 3-term progressions. We present an analogous result for longer progressions by combining a quantified version of the relative Szemerédi theorem given by Conlon, Fox and Zhao with Henriot's estimates of the enveloping sieve weights. This is joint work with Luka Rimanic.

Cyclotomic factors of Serre's polynomials

Florian Luca (U. Witwatersrand)

Consider the family of polynomials $P_m(X) \in \mathbb{Q}[X]$ given by

$$\prod_{m\geq 1}(1-q^m)^{-z} = \sum_{m\geq 0} P_m(z)q^m.$$

These polynomials have deep connections with the theory of partition numbers and the Ramanujan τ-function. They appeared for the first time in work of Newman

1955, and were used by Serre in his 1985 work on the lacunarity of the powers of the Dedekind eta function. They can also be given recursively as $P_0(X) = 1$ and

$$P_m(X) = \frac{X}{m}\left(\sum_{k=1}^{m} \sigma(k) P_{m-k}(X)\right).$$

It is easy to see that $P_m(X)$ has no positive real roots. Further, by the Euler pentagonal formula, it follows that $X + 1 \mid P_m(X)$ for infinitely many m. We ask whether $P_m(X)$ can have other roots of unity except -1. We prove that this is never the case, namely that if ζ is a root of unity of order $N \geq 3$ and $m \geq 1$, then $P_m(\zeta) \neq 0$. The proof uses basic facts about finite fields and a bit of analytic number theory

Joint work with Bernhard Heim and Markus Neuhauser.

Sidon sets and bases

Sandor Kiss (Budapest U. of Technology and Economics)

Abstract: Let $h \geq 2$ be an integer. We say that a set A of positive integers is an asymptotic basis of order h if every large enough positive integer can be represented as the sum of h terms from A. A set of positive integers A is called a Sidon set if all the sums $a + b$ with $a, b \in A$, $a \leq b$ are distinct. In my talk I am going to speak about Sidon sets which are asymptotic bases of order 4, and a related problem about multiplicative Sidon sets, which is a joint work with Javier Cilleruelo.

Counting configuration–free sets in groups

Lluis Vena (U. of Amsterdam)

Abstract: By combining the hypergraph containers methodology joint with arithmetic removal lemmas, we are able to provide a unified framework to asymptotically count the number of sets, with a given cardinality, free of certain configurations. Several applications involving linear configurations are described, as well as some applications in the random sparse setting.

A tale of two offices

Pablo Fernández (U. Autónoma de Madrid)

Abstract: In this talk, we will describe a brief (but extremely pleasant) collaboration with Javier Cilleruelo.
A random walker starts moving up and right in the integer lattice, with probabilities α and $1 - \alpha$, respectively, and starting from the origin.
What is the (asymptotic) proportion of time the random walker remains visible from the origin? Does it depend on α? Some other questions about random walks in the lattice will be discussed.